

Corrigendum to “Flavour Covariant Transport Equations: an Application to Resonant Leptogenesis”

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Abstract

We amend the incorrect discussion in Nucl. Phys. B **886** (2014) 569 [1] concerning the numerical examples considered there. In particular, we discuss the viability of minimal radiative models of Resonant Leptogenesis and prove that no asymmetry can be generated at $\mathcal{O}(h^4)$ in these scenarios. We present a minimal modification of the model considered in [1], where electroweak-scale right-handed Majorana neutrinos can easily accommodate both successful leptogenesis and observable signatures at Lepton Number and Flavour Violation experiments. The importance of the fully flavour-covariant rate equations, as developed in [1], for describing accurately the generation of the lepton asymmetry is reconfirmed.

In this note, we discuss the viability of minimal radiative Resonant Leptogenesis (RL) scenarios. In these models, the mass splitting between the right-handed (RH) heavy Majorana neutrinos, which can explain the observed baryon asymmetry of the Universe, as well as the low-energy neutrino data, is generated *entirely* by the renormalization-group (RG) running from some high mass scale μ_X (of the order of the grand unification scale) down to the relevant heavy-neutrino mass scale m_N . We take the latter to be of the order of the electroweak scale, so that it can be tested in current and future experiments. This is the scenario used in the numerical examples of [1]. Here we amend some incorrect discussions presented in Sections 5 and 6 of this article. The incorrectness of some of the numerical results given there is related to the usage, in the numerical analysis, of the incorrect formulae (2.9) and (2.13) of [2], reported in (5.10) of [1].

1. No-go theorem for minimal radiative RL at $\mathcal{O}(h^4)$

The relevant heavy-neutrino Lagrangian is given by

$$-\mathcal{L}_N = h_l \alpha \bar{L}^l \tilde{\Phi} N_{R,\alpha} + \frac{1}{2} \bar{N}_{R,\alpha}^C [M_N]^{\alpha\beta} N_{R,\beta} + \text{H.c.} , \quad (1)$$

where $\tilde{\Phi} = i\sigma_2\Phi^*$ is the isospin conjugate of the Higgs doublet Φ and the superscript C denotes charge conjugation. In minimal radiative scenarios, the masses of these heavy neutrinos N_α ($\alpha = 1, 2, 3$) are assumed to be degenerate at a high scale $\mu_X \sim 10^{16}$ GeV, thanks to an approximate $O(3)$ symmetry: $\mathbf{M}_N(\mu_X) = m_N \mathbf{1}_3$. At the scale m_N , relevant for leptogenesis, the mass matrix \mathbf{M}_N is obtained by the RG evolution from μ_X to m_N :

$$\mathbf{M}_N = m_N \mathbf{1}_3 + \Delta \mathbf{M}_N^{\text{RG}}, \quad (2)$$

where, in the minimal radiative RL scenario, $\Delta \mathbf{M}_N^{\text{RG}}$ is taken to be the only $O(3)$ -breaking correction to the mass matrix and is given by

$$\Delta \mathbf{M}_N^{\text{RG}} \simeq -\frac{m_N}{8\pi^2} \ln\left(\frac{\mu_X}{m_N}\right) \text{Re}[\mathbf{h}^\dagger(\mu_X)\mathbf{h}(\mu_X)]. \quad (3)$$

However, as we are going to show below, this minimal scenario is not viable at $\mathcal{O}(h^4)$, because of the following *no-go theorem for minimal radiative RL at $\mathcal{O}(h^4)$* . The RH neutrino mass matrix given by (2) and (3) is real and symmetric and, as long as we are in the perturbative regime $|\Delta \mathbf{M}_N^{\text{RG}}|_{\alpha\beta}/m_N \ll 1$, it can be diagonalized with positive eigenvalues by a real orthogonal matrix $\mathbf{O} \in O(3) \subset U(3)$:

$$\mathbf{M}_N = \mathbf{O} \widehat{\mathbf{M}}_N \mathbf{O}^\text{T}, \quad (4)$$

where the caret ($\widehat{}$) denotes the mass eigenbasis. At leading order, i.e. $\mathcal{O}(h^2)$, the Yukawa couplings in (3) can be taken at the scale m_N . Since \mathbf{O} is real and orthogonal, both $\mathbf{O}^\text{T} \Delta \mathbf{M}_N^{\text{RG}} \mathbf{O}$ and

$$\text{Re}(\widehat{\mathbf{h}}^\dagger \widehat{\mathbf{h}}) = \text{Re}[(\mathbf{O}^\text{T} \mathbf{h}^\dagger)(\mathbf{h} \mathbf{O})] = \mathbf{O}^\text{T} \text{Re}(\mathbf{h}^\dagger \mathbf{h}) \mathbf{O} \propto \mathbf{O}^\text{T} \Delta \mathbf{M}_N^{\text{RG}} \mathbf{O} \quad (5)$$

are also separately diagonal. On the other hand, the leptonic asymmetry $\varepsilon_{l\alpha}$ in the decay $N_\alpha \rightarrow L_l \Phi$ is proportional to the quantity (cf. (A.2) in [1])

$$\text{Im}[\widehat{h}_{l\alpha}^* \widehat{h}_{l\beta} (\widehat{h}^\dagger \widehat{h})_{\alpha\beta}] + \frac{m_{N,\alpha}}{m_{N,\beta}} \text{Im}[\widehat{h}_{l\alpha}^* \widehat{h}_{l\beta} (\widehat{h}^\dagger \widehat{h})_{\beta\alpha}] = 2 \text{Im}[\widehat{h}_{l\alpha}^* \widehat{h}_{l\beta}] \text{Re}[(\widehat{h}^\dagger \widehat{h})_{\alpha\beta}] + \mathcal{O}(h^6), \quad (6)$$

where $m_{N,\alpha}$ is the physical mass of N_α and $\alpha \neq \beta$. Therefore, the leptonic asymmetry $\varepsilon_{l\alpha} \propto \text{Re}[(\widehat{h}^\dagger \widehat{h})_{\alpha\beta}]$, being proportional to the off-diagonal entries of a diagonal matrix, vanishes identically at $\mathcal{O}(h^4)$ in minimal radiative models, where no other source of $O(3)$ flavour breaking is present. \square

2. A next-to-minimal radiative RL model

To avoid the no-go theorem of suppressed leptonic asymmetries, as derived in Section 1, we proceed differently from [1, 2]. We include a new source of flavour breaking $\Delta \mathbf{M}_N$, which is *not* aligned with $\text{Re}(\mathbf{h}^\dagger \mathbf{h})$ at the input scale μ_X . More explicitly, the heavy-neutrino mass matrix takes on the following form:

$$\mathbf{M}_N = m_N \mathbf{1} + \Delta \mathbf{M}_N + \Delta \mathbf{M}_N^{\text{RG}}. \quad (7)$$

For the purposes of this note, we consider a minimal breaking matrix $\Delta\mathbf{M}_N$ of the form

$$\Delta\mathbf{M}_N = \begin{pmatrix} \Delta M_1 & 0 & 0 \\ 0 & \Delta M_2/2 & 0 \\ 0 & 0 & -\Delta M_2/2 \end{pmatrix}, \quad (8)$$

where ΔM_2 is needed to make the light-neutrino mass matrix rank-2, thus allowing us to fit successfully the low-energy neutrino data. On the other hand, ΔM_1 governs the mass difference between N_1 and $N_{2,3}$, and its inclusion is sufficient to obtain successful leptogenesis.

In order to protect the lightness of the left-handed neutrinos in a technically natural manner, we consider an RL_τ model that possesses a leptonic symmetry $U(1)_l$. In this scenario, the Yukawa couplings $h_l\alpha$ have the following structure [3, 4]:

$$\mathbf{h} = \begin{pmatrix} 0 & a e^{-i\pi/4} & a e^{i\pi/4} \\ 0 & b e^{-i\pi/4} & b e^{i\pi/4} \\ 0 & c e^{-i\pi/4} & c e^{i\pi/4} \end{pmatrix} + \delta\mathbf{h}, \quad (9)$$

where, in order to protect the τ asymmetry from excessive washout and at the same time guarantee observable effects in low-energy neutrino experiments, we take $|c| \ll |a|, |b| \approx 10^{-3} - 10^{-2}$. The leptonic flavour-symmetry-breaking matrix is taken to be

$$\delta\mathbf{h} = \begin{pmatrix} \epsilon_e & 0 & 0 \\ \epsilon_\mu & 0 & 0 \\ \epsilon_\tau & 0 & 0 \end{pmatrix}. \quad (10)$$

To leading order in the symmetry-breaking parameters of $\Delta\mathbf{M}_N$ and $\delta\mathbf{h}$, the tree-level light-neutrino mass matrix is given by the seesaw formula

$$\mathbf{M}_\nu \simeq -\frac{v^2}{2}\mathbf{h}\mathbf{M}_N^{-1}\mathbf{h}^\top \simeq \frac{v^2}{2m_N} \begin{pmatrix} \frac{\Delta m_N}{m_N}a^2 - \epsilon_e^2 & \frac{\Delta m_N}{m_N}ab - \epsilon_e\epsilon_\mu & -\epsilon_e\epsilon_\tau \\ \frac{\Delta m_N}{m_N}ab - \epsilon_e\epsilon_\mu & \frac{\Delta m_N}{m_N}b^2 - \epsilon_\mu^2 & -\epsilon_\mu\epsilon_\tau \\ -\epsilon_e\epsilon_\tau & -\epsilon_\mu\epsilon_\tau & -\epsilon_\tau^2 \end{pmatrix}, \quad (11)$$

where

$$\Delta m_N \equiv 2[\Delta M_N]_{23} + i([\Delta M_N]_{33} - [\Delta M_N]_{22}) = -i\Delta M_2 \quad (12)$$

and we have neglected subdominant terms $\frac{\Delta m_N}{m_N}c \times (a, b, c)$. Assuming a particular mass hierarchy between the light-neutrino masses m_{ν_i} and for given values of the CP phases $\delta, \varphi_{1,2}$, we determine the following model parameters appearing in the Yukawa coupling matrix (9):

$$\begin{aligned} a^2 &= \frac{2m_N}{v^2} \left(M_{\nu,11} - \frac{M_{\nu,13}^2}{M_{\nu,33}} \right) \frac{m_N}{\Delta m_N}, & b^2 &= \frac{2m_N}{v^2} \left(M_{\nu,22} - \frac{M_{\nu,23}^2}{M_{\nu,33}} \right) \frac{m_N}{\Delta m_N}, \\ \epsilon_e^2 &= -\frac{2m_N}{v^2} \frac{M_{\nu,13}^2}{M_{\nu,33}}, & \epsilon_\mu^2 &= -\frac{2m_N}{v^2} \frac{M_{\nu,23}^2}{M_{\nu,33}}, & \epsilon_\tau^2 &= -\frac{2m_N}{v^2} M_{\nu,33}. \end{aligned} \quad (13)$$

Parameters	BP1	BP2	BP3
m_N	120 GeV	400 GeV	5 TeV
c	2×10^{-6}	2×10^{-7}	2×10^{-6}
$\Delta M_1/m_N$	-5×10^{-6}	-3×10^{-5}	-4×10^{-5}
$\Delta M_2/m_N$	$(-1.59 - 0.47i) \times 10^{-8}$	$(-1.21 + 0.10i) \times 10^{-9}$	$(-1.46 + 0.11i) \times 10^{-8}$
a	$(5.54 - 7.41i) \times 10^{-4}$	$(4.93 - 2.32i) \times 10^{-3}$	$(4.67 - 4.33i) \times 10^{-3}$
b	$(0.89 - 1.19i) \times 10^{-3}$	$(8.04 - 3.79i) \times 10^{-3}$	$(7.53 - 6.97i) \times 10^{-3}$
ϵ_e	$3.31i \times 10^{-8}$	$5.73i \times 10^{-8}$	$2.14i \times 10^{-7}$
ϵ_μ	$2.33i \times 10^{-7}$	$4.30i \times 10^{-7}$	$1.50i \times 10^{-6}$
ϵ_τ	$3.50i \times 10^{-7}$	$6.39i \times 10^{-7}$	$2.26i \times 10^{-6}$

Table 1: The numerical values of the free (m_N , c , $\Delta M_{1,2}$) and derived parameters (a , b , $\epsilon_{e,\mu,\tau}$) in the RL_τ model for three chosen benchmark points.

Therefore, the Yukawa coupling matrix (9) in the RL_τ model can be completely fixed in terms of the heavy-neutrino mass scale m_N and the input parameters c and ΔM_2 . Notice that, whereas (11) and (13) coincide formally with the corresponding formulae in [1, 2], the latter are incorrect for the model considered therein.

We amend the three benchmark points considered in [1] as detailed in Table 1. The input parameters ΔM_1 and c are easily chosen such that leptogenesis is successful. But ΔM_2 has been tuned here in order to reproduce *exactly* the predictions for the Lepton Number and Flavour Violation (LNV and LFV) observables discussed in [1]. In particular, Table 4 of [1] is unaltered, thus confirming the observable effects in LNV and LFV experiments predicted by this class of models, while simultaneously allowing for successful leptogenesis. The CP phases in the light neutrino sector have been chosen as $\varphi_1 = -\pi$ and $\varphi_2 = \delta = 0$.

The discussion in Section 5.3.2 of [1], concerning the approximate analytic solution for the charged-lepton decoherence effect, also requires modifications. In particular, some of the approximations adopted there are no longer valid. In light of this, (5.22) of [1] becomes

$$\begin{aligned}
\frac{d}{dz}[\delta\hat{\eta}^L]_{lm} &= \frac{z^3 K_1(z)}{2} \left(\sum_\alpha [\hat{\eta}^N]_{\alpha\alpha} [\delta\hat{K}_{L\Phi}^N]_{lm\alpha\alpha} - \frac{1}{3} \left\{ \delta\hat{\eta}^L, \hat{K}^{\text{eff}} \right\}_{lm} \right. \\
&\quad \left. - \frac{2}{3} [\delta\hat{\eta}^L]_{kn} [\hat{K}_-]_{nklm} - \frac{2}{3} \left\{ \delta\hat{\eta}^L, \hat{K}_{\text{dec}} \right\}_{lm} + [\delta\hat{K}_{\text{dec}}^{\text{back}}]_{lm} \right), \quad (14)
\end{aligned}$$

where $\{, \}$ denotes anti-commutators in flavour space and we need to introduce also the K -factor $[\hat{K}_-]_{nklm} = \kappa [\hat{\gamma}_{L\bar{e}\Phi\bar{e}}^{L\Phi} - \hat{\gamma}_{L\Phi}^{L\Phi}]_{nklm}$, which is no longer subdominant. Correspondingly, (5.26) of [1] is modified to

$$\frac{1}{3} \left\{ \delta\hat{\eta}^L, \hat{K}^{\text{eff}} + 2\hat{K}_{\text{dec}} \right\}_{lm} - [\delta\hat{K}_{\text{dec}}^{\text{back}}]_{lm} + \frac{2}{3} [\delta\hat{\eta}^L]_{kn} [\hat{K}_-]_{nklm} \simeq \frac{\hat{\epsilon}_{lm}}{z}. \quad (15)$$

It is not easy to perform further approximations in this equation, and therefore, it is convenient to solve numerically the linear system (15) for the variables $[\delta\hat{\eta}^L]_{lm}$. We then obtain the

semi-analytic contribution of mixing and charged-lepton (de)coherence to the asymmetry in the strong-washout regime

$$\delta\hat{\eta}^L \supset \delta\hat{\eta}_{\text{mix}}^L + \delta\hat{\eta}_{\text{dec}}^L \simeq \sum_l [\delta\hat{\eta}^L]_l, \quad (16)$$

where the diagonal asymmetries $[\delta\hat{\eta}^L]_l$ are obtained by solving the linear system (15).

Finally, Figs. 8–11 of [1] are modified too. The amended numerical results are shown in Figs. figN-fig7 of this note. The main qualitative difference with respect to those given in Section 6.2 of [1] is that the contribution of the charged-lepton off-diagonal number densities now *suppresses* the total asymmetry for the three benchmark points considered here, rather than enhancing it, as in Figs. 10–11 of [1]. Nevertheless, successful leptogenesis is still comfortably realized. Thus, we may conclude that the salient features discussed in Sections 5 and 6 of [1] remain valid, namely the joint possibility of successful leptogenesis and observable signatures in LNV and LFV experiments. Moreover, as is evidenced by the disparity between the asymmetries predicted by the partially flavour-off-diagonal treatments in Figs. 3 and 4, the use of fully flavour-covariant rate equations, as developed in [1], remains of paramount importance for obtaining accurate quantitative predictions in this class of models.

Acknowledgements

We thank Jonathan Da Silva for pointing out the incorrectness of (2.13) in [2], reported as (5.10) in [1]. The work of P.S.B.D. and A.P. is supported by the Lancaster-Manchester-Sheffield Consortium for Fundamental Physics under STFC grant ST/L000520/1. The work of P.M. is supported by a University Foundation Fellowship (TUFF) from the Technische Universität München and the Deutsche Forschungsgemeinschaft (DFG) cluster of excellence Origin and Structure of the Universe. The work of D.T. has been supported by a fellowship of the EPS Faculty of the University of Manchester.

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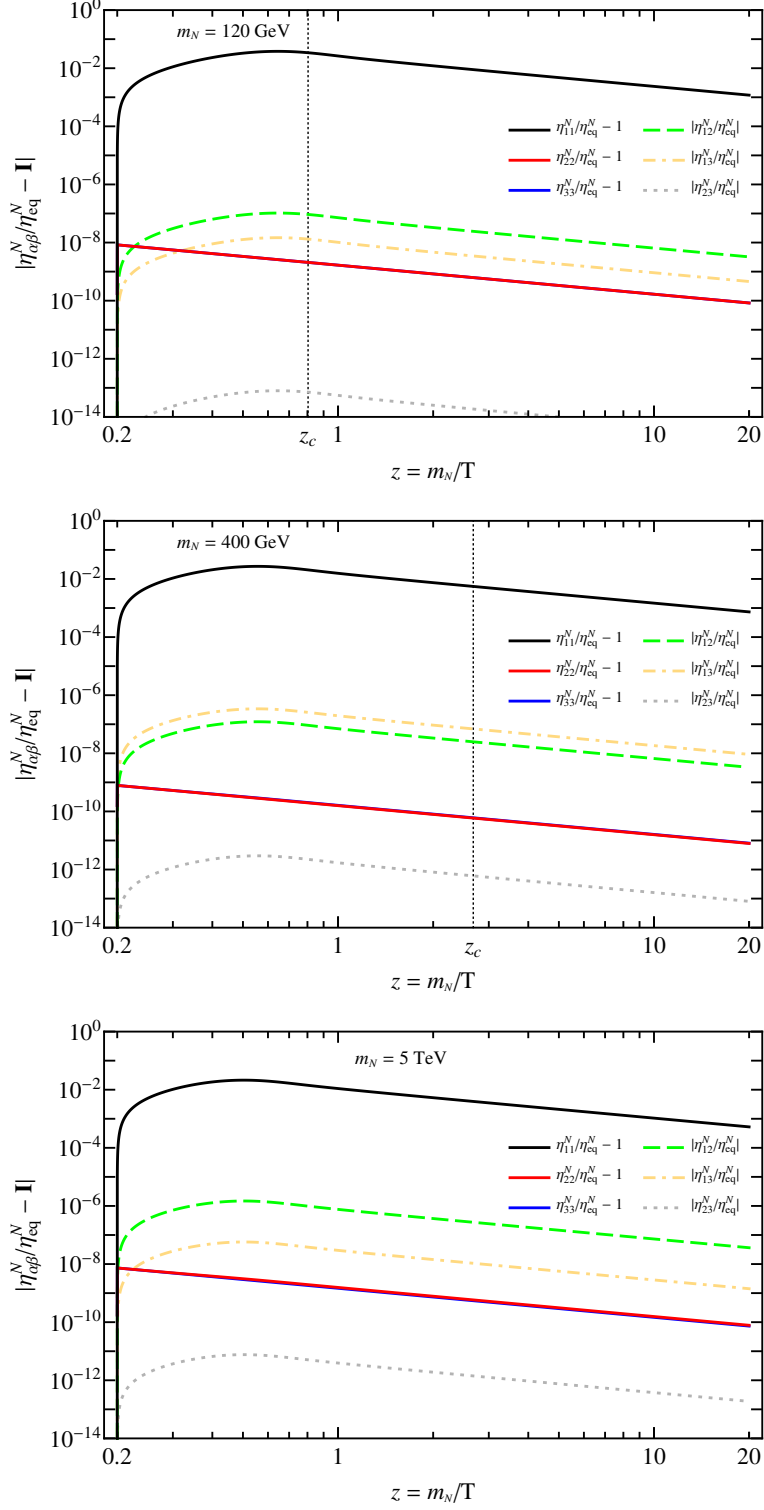


Figure 1: The deviation of the heavy-neutrino number densities $\hat{\eta}_{\alpha\beta}^N = \hat{\eta}_{\alpha\beta}^N/\eta_{\text{eq}}^N - \delta_{\alpha\beta}$ from their equilibrium values for the three benchmark points given in Table 1. The different lines show the evolution of the diagonal (solid lines) and off-diagonal (dashed lines) number densities in the fully flavour-covariant formalism. The numerical values of $\hat{\eta}_{22}^N$ and $\hat{\eta}_{33}^N$ coincide with each other in all three cases.

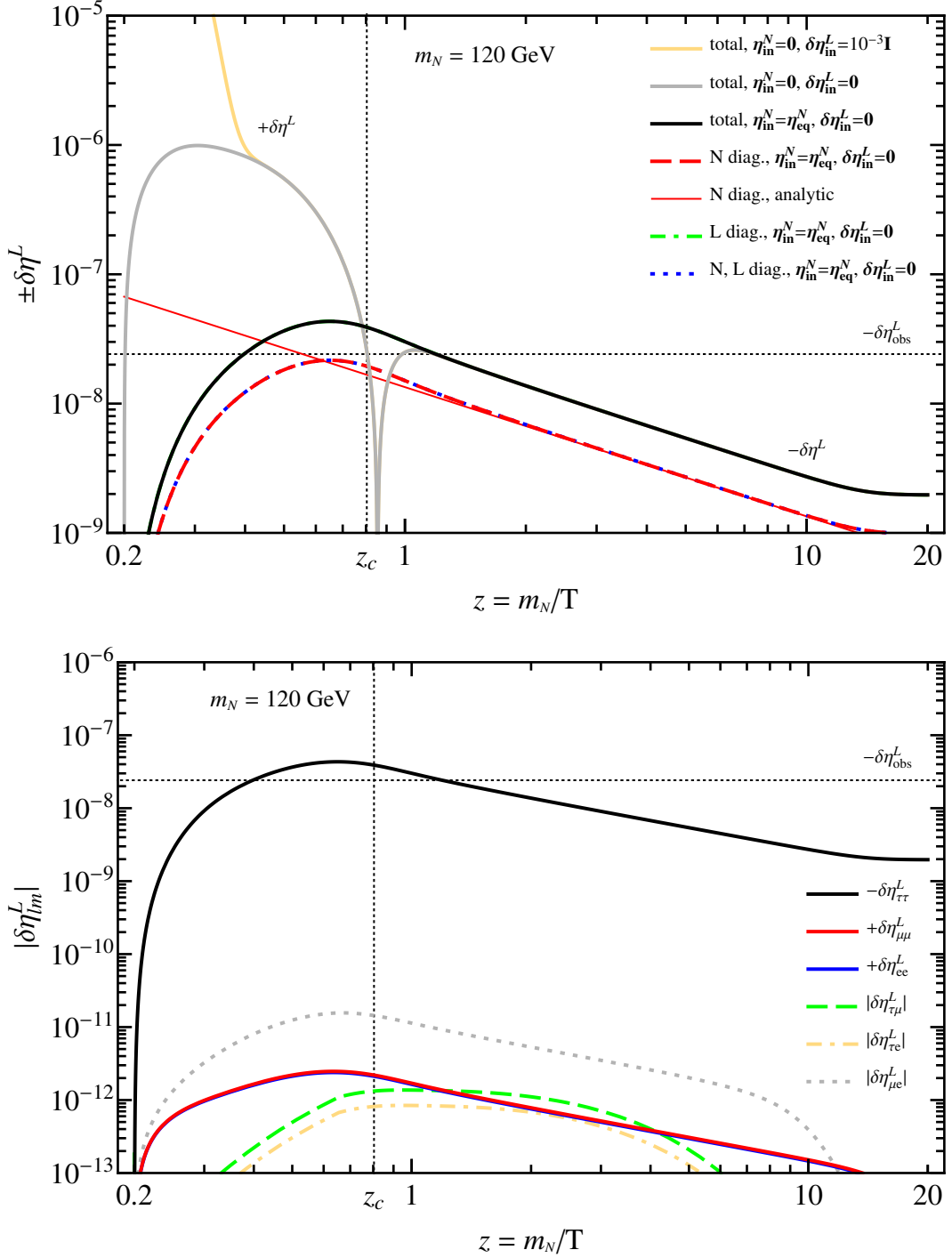


Figure 2: Lepton flavour asymmetries as predicted by the BP1 parameters given in Table 1. The top panel shows the comparison between the total asymmetry obtained using the fully flavour-covariant formalism (thick solid lines, with different initial conditions) with those obtained using the flavour-diagonal formalism (dashed lines). Also shown (thin solid line) is the semi-analytic result (16). The bottom panel shows the diagonal (solid lines) and off-diagonal (dashed lines) elements of the total lepton number asymmetry matrix in the fully flavour-covariant formalism. $\delta \hat{\eta}_{ee}^L$ and $\delta \hat{\eta}_{\mu\mu}^L$ are coincident. For details, see the text and [1].

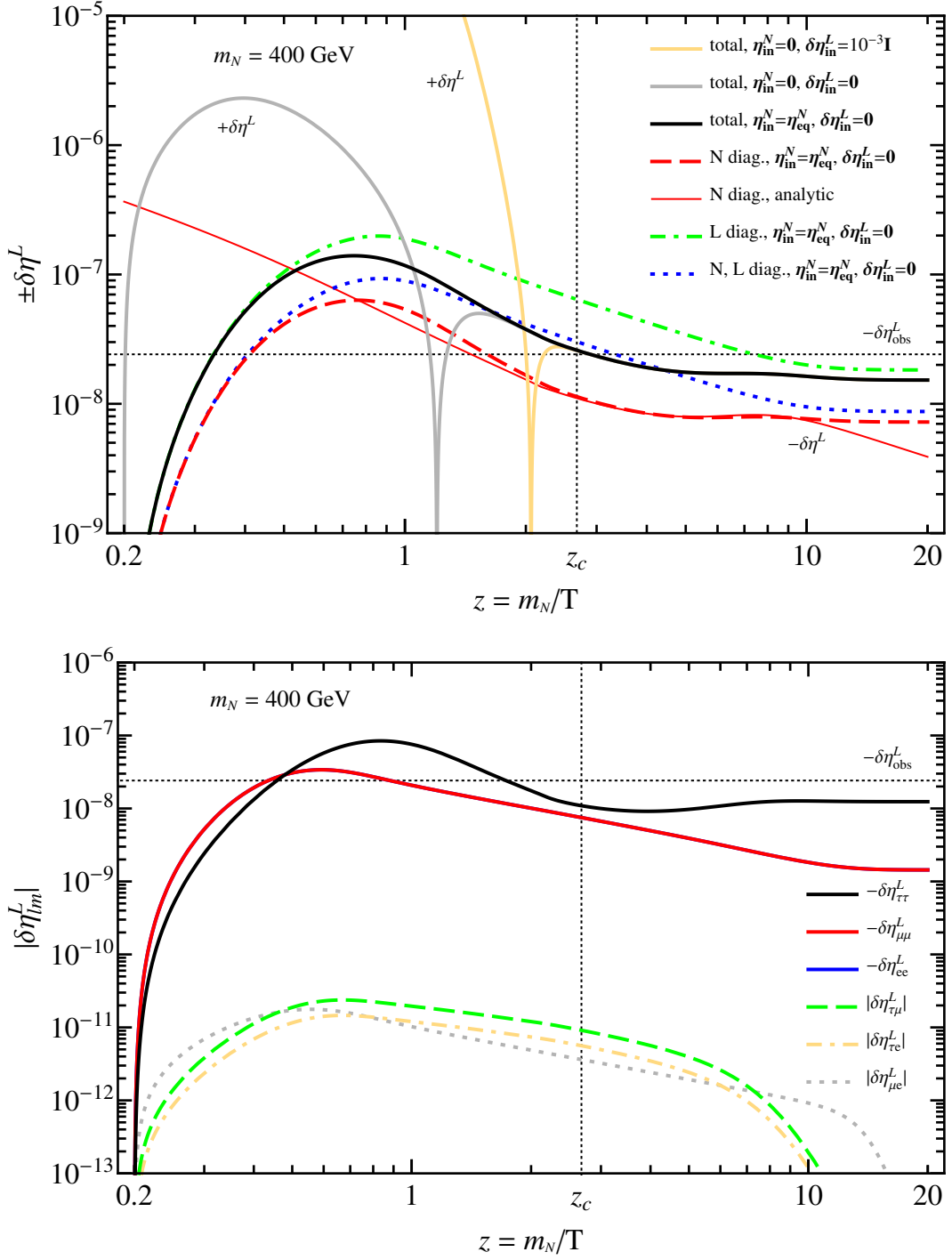


Figure 3: Lepton flavour asymmetries as predicted by the BP2 RL_τ model parameters given in Table 1. The labels are the same as in Figure 2. $\delta\hat{\eta}_{ee}^L$ and $\delta\hat{\eta}_{\mu\mu}^L$ are coincident.

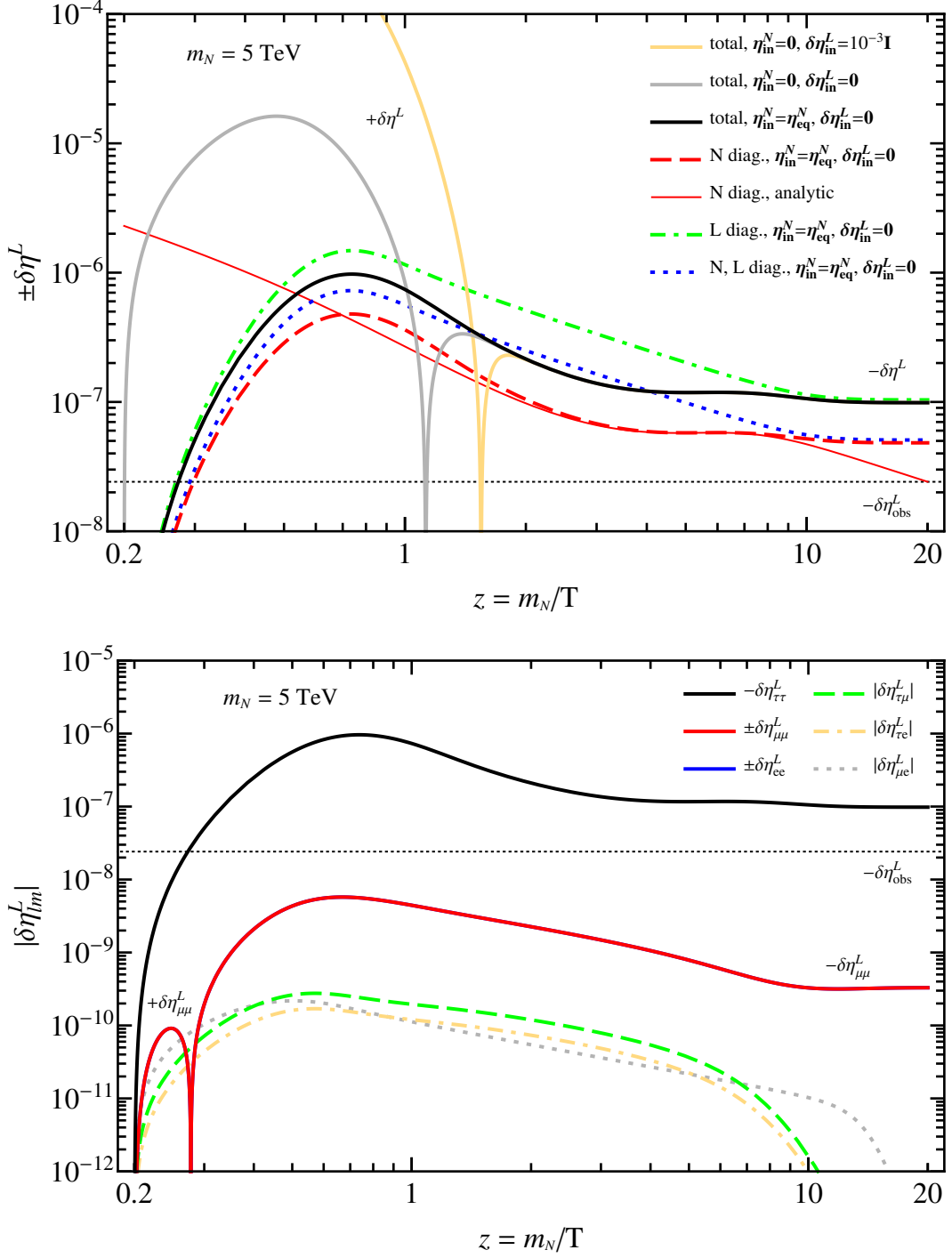


Figure 4: Lepton flavour asymmetries as predicted by the BP3 RL_τ model parameters given in Table 1. The labels are the same as in Figure 2. $\delta\hat{\eta}_{ee}^L$ and $\delta\hat{\eta}_{\mu\mu}^L$ are coincident.